Models of Direct Time-of-Flight Sensor Precision with Implications for On-Chip Histogram Processing

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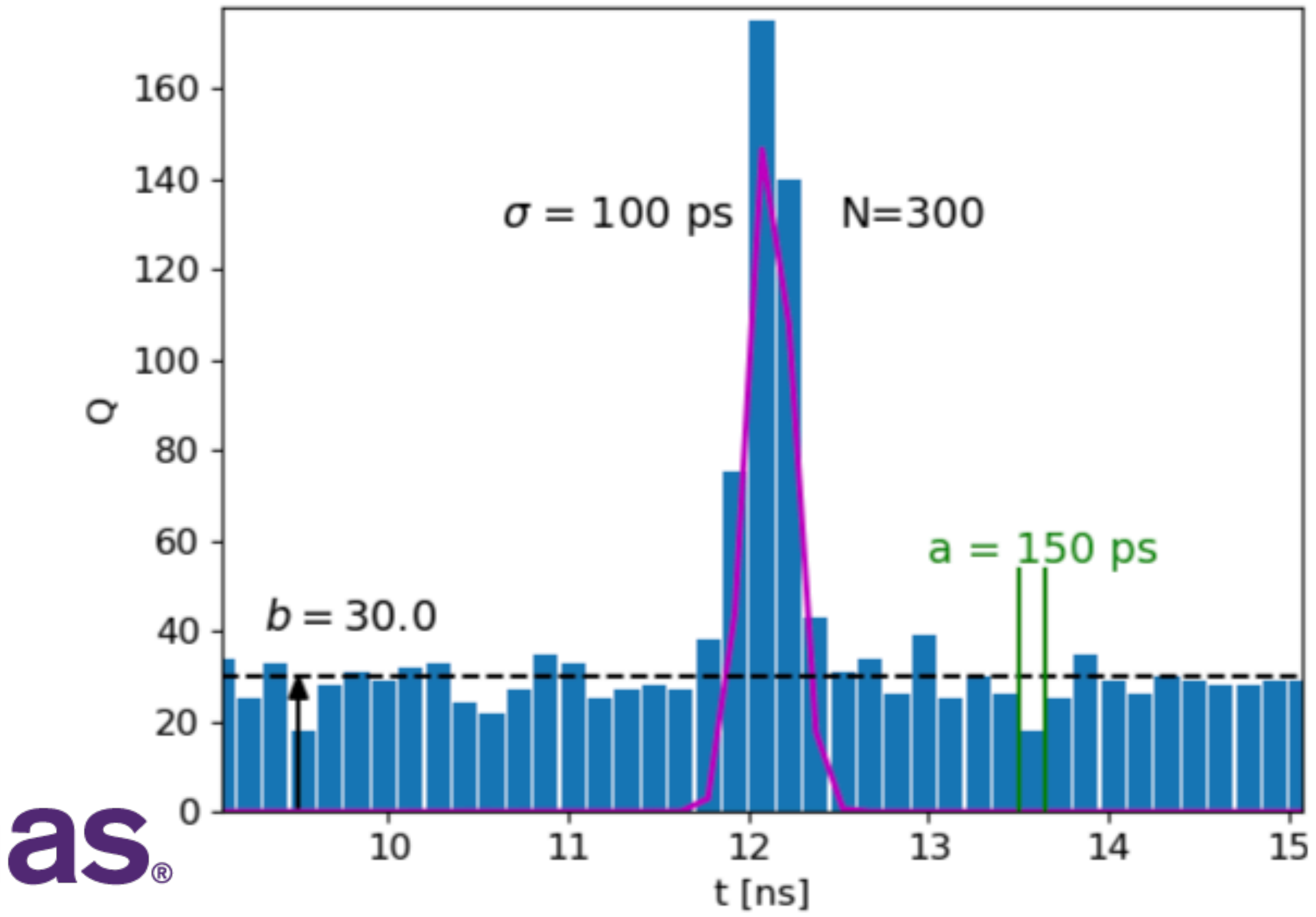
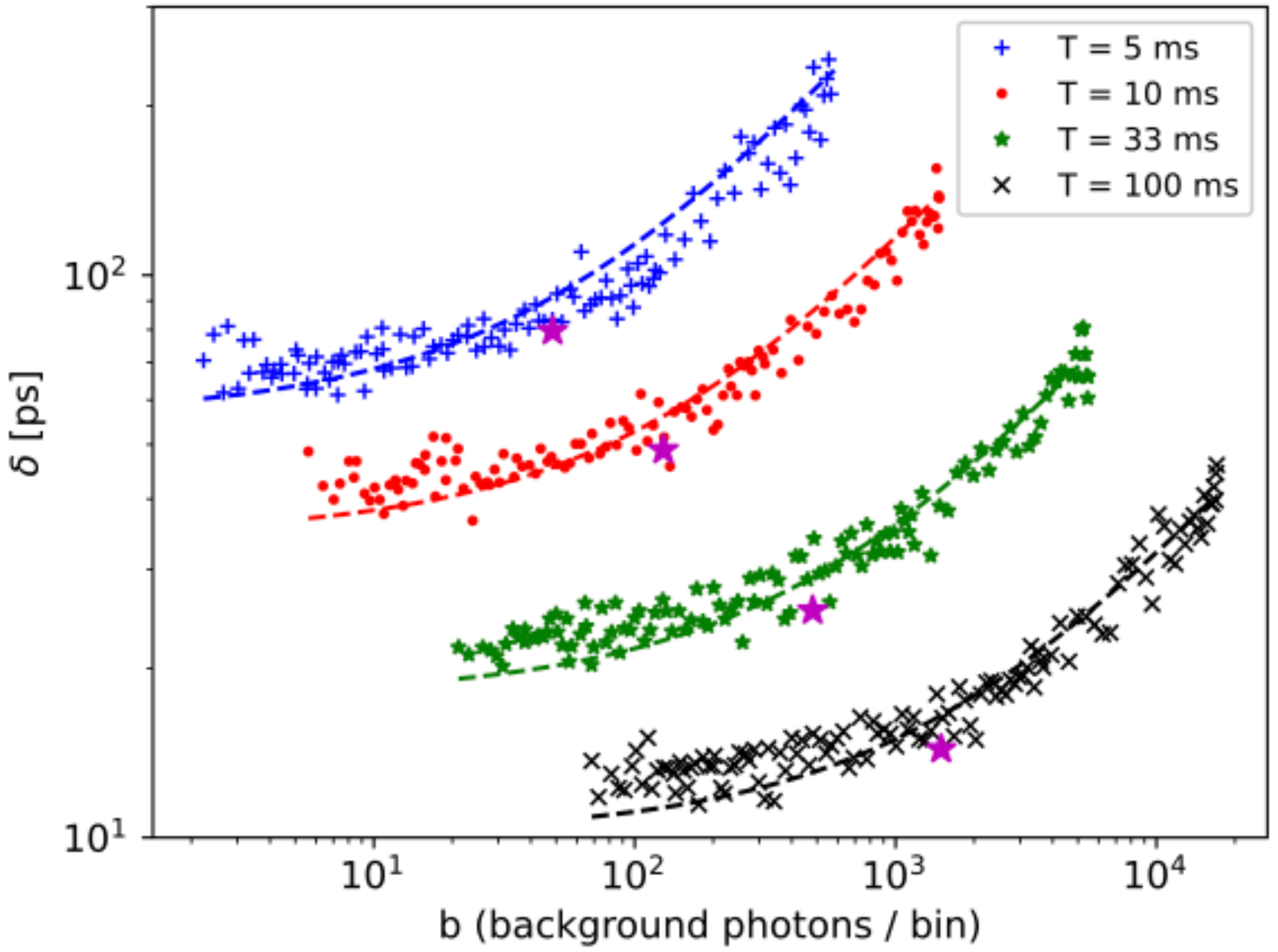
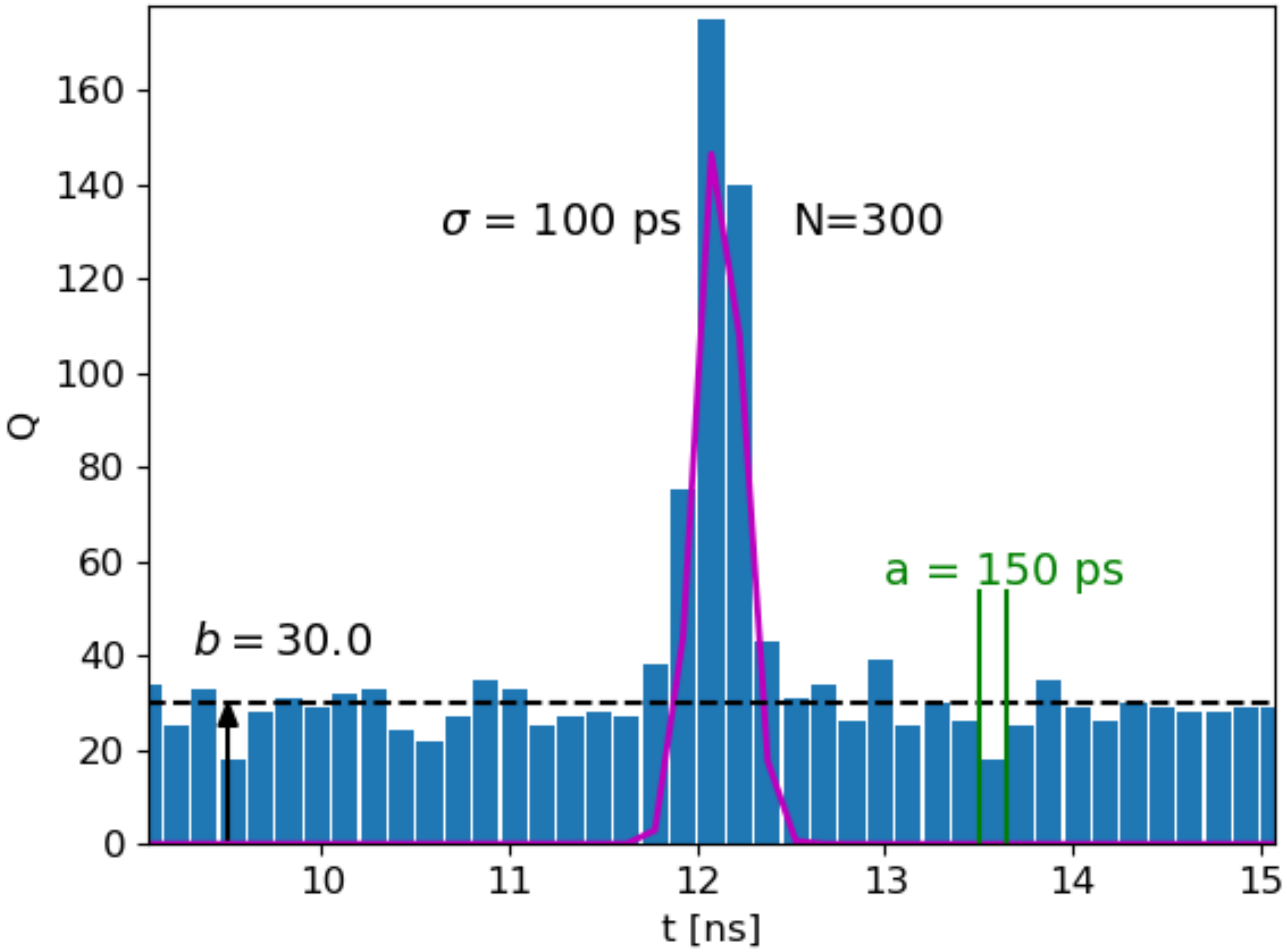
2022/05/20





Objectives

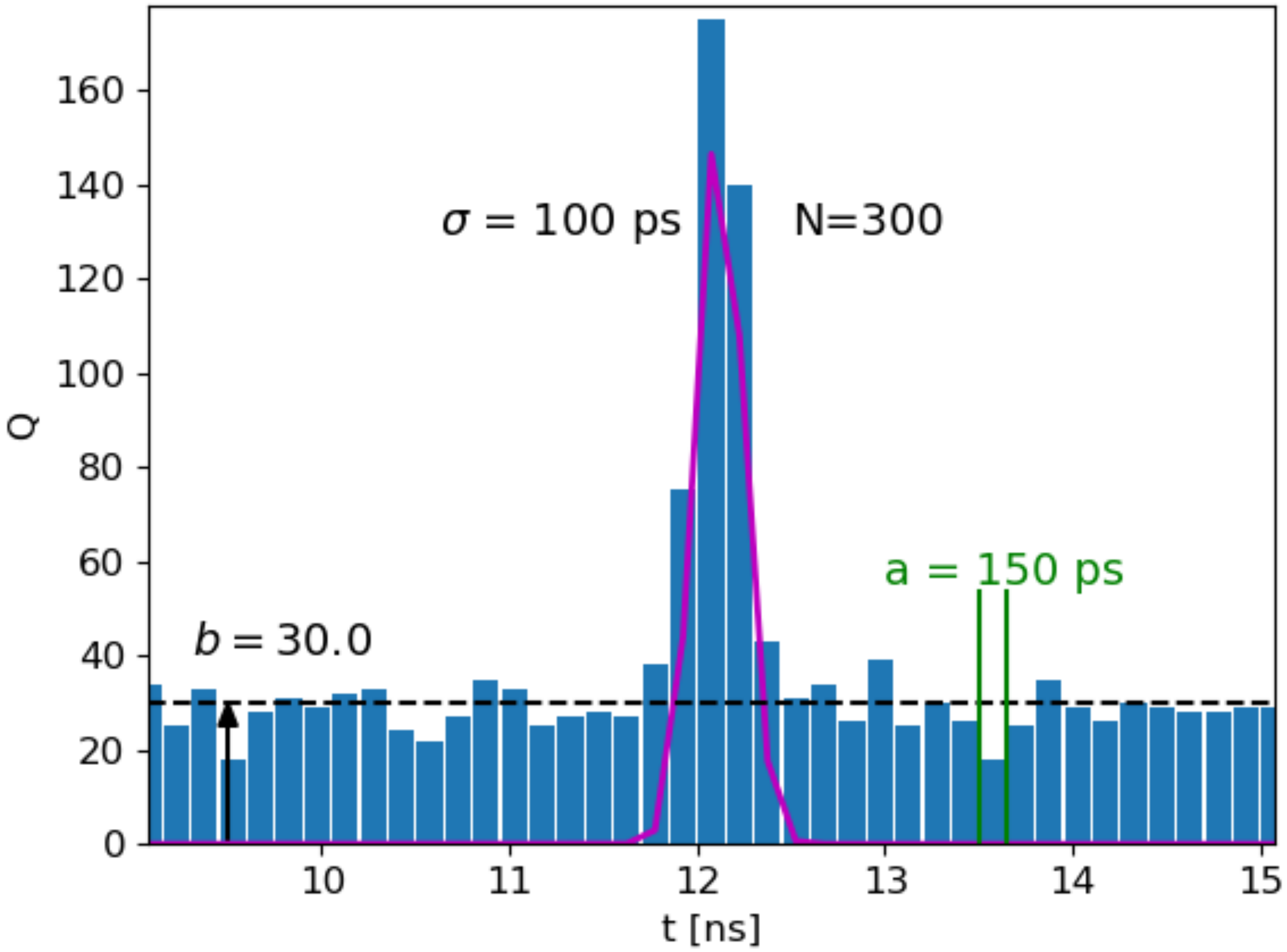
- Model depth precision from histogram signals and sensor design parameters.
- Use models to guide dToF design and dynamic configuration.
    - Design: optimal TDC bin size?
    - Configuration: exposure time for 2 mm precision?



Model

Symbol	Description	Unit
N	Total signal photons collected in peak	-
b	Background photons per TDC bin	-
a	TDC bin width	s
σ	Instrument response function (IRF). Includes laser pulse width, SPAD timing jitter, etc.	s
δ	Resulting depth precision (standard deviation of return time)	s

- Gaussian instrument response



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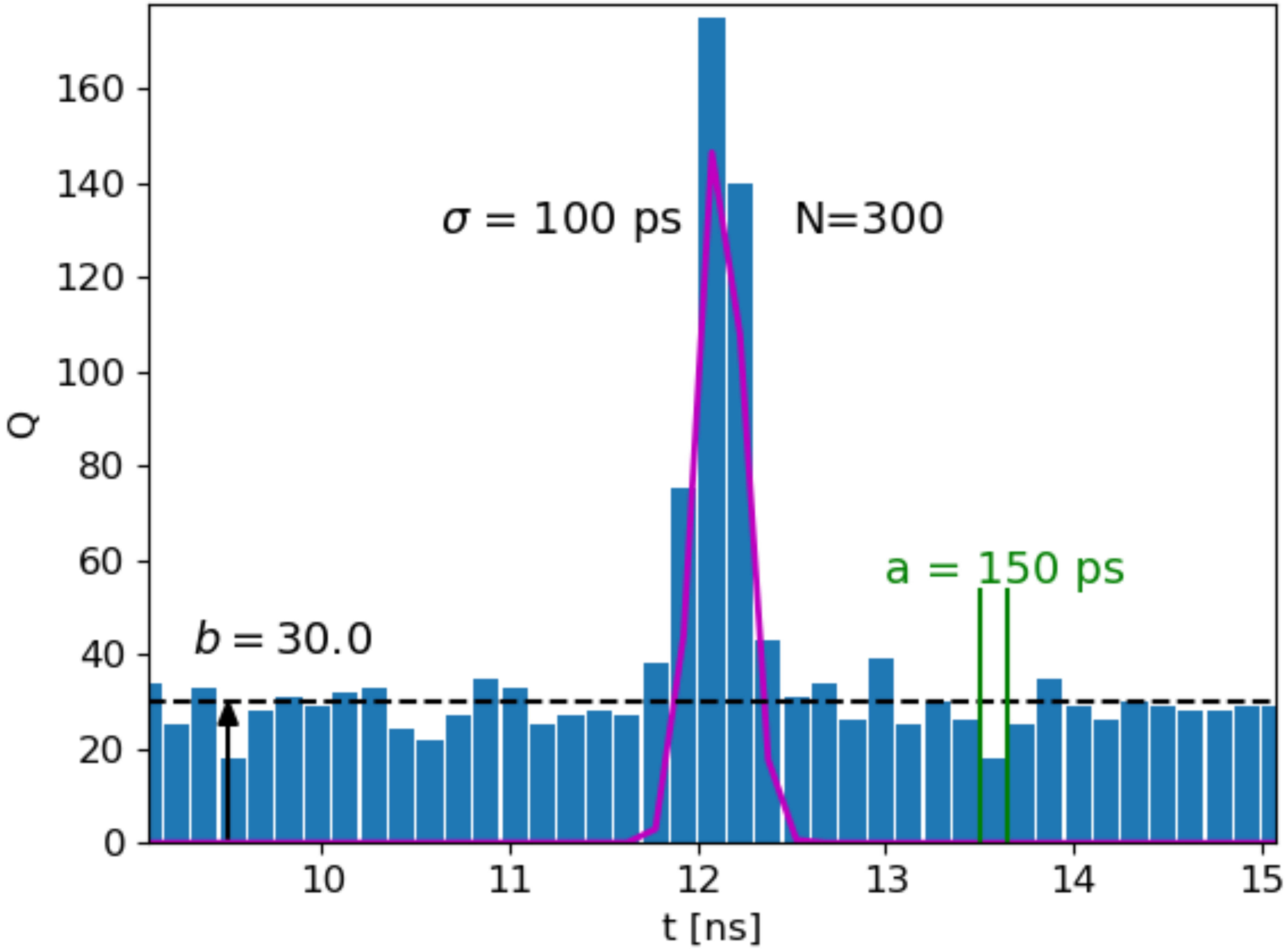
- Simple for modularity
- Optical and scene parameters can be studied based on impact to N, b
  - Example. Lens f-number: N ~ $1/(f/\#)^2$, b ~ $1/(f/\#)^2$



Analytical Models

Symbol	Description
N	signal photons
b	background/bin
a	bin width
σ	timing jitter
δ	precision

1) Fundamental limit

$$\delta = \frac{\sigma}{\sqrt{N}}$$

2) Thompson

$$\delta = \sqrt{\frac{\sigma^2 + a^2/12}{N} + \frac{4\sqrt{\pi}\sigma^3 b}{aN^2}} = \frac{\sigma}{\sqrt{N}}\sqrt{1 + \frac{1}{12}\left(\frac{a}{\sigma}\right)^2 + 4\sqrt{\pi}\left(\frac{\sigma}{a}\right)\left(\frac{b}{N}\right)}$$

3) Cramér-Rao Lower Bound (CRB): information limit from Gaussian PDF,
most accurate, requires integration over TDC bin

L. J. Koerner, "Models of Direct Time-of-Flight Sensor Precision That Enable Optimal Design and Dynamic Configuration," 10.1109/TIM.2021.3073684

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R. E. Thompson, D. R. Larson, and W. W. Webb, “Precise nanometer localization analysis for individual fluorescent probes,” Biophys. J., 10.1016/S0006-3495(02)75618-X

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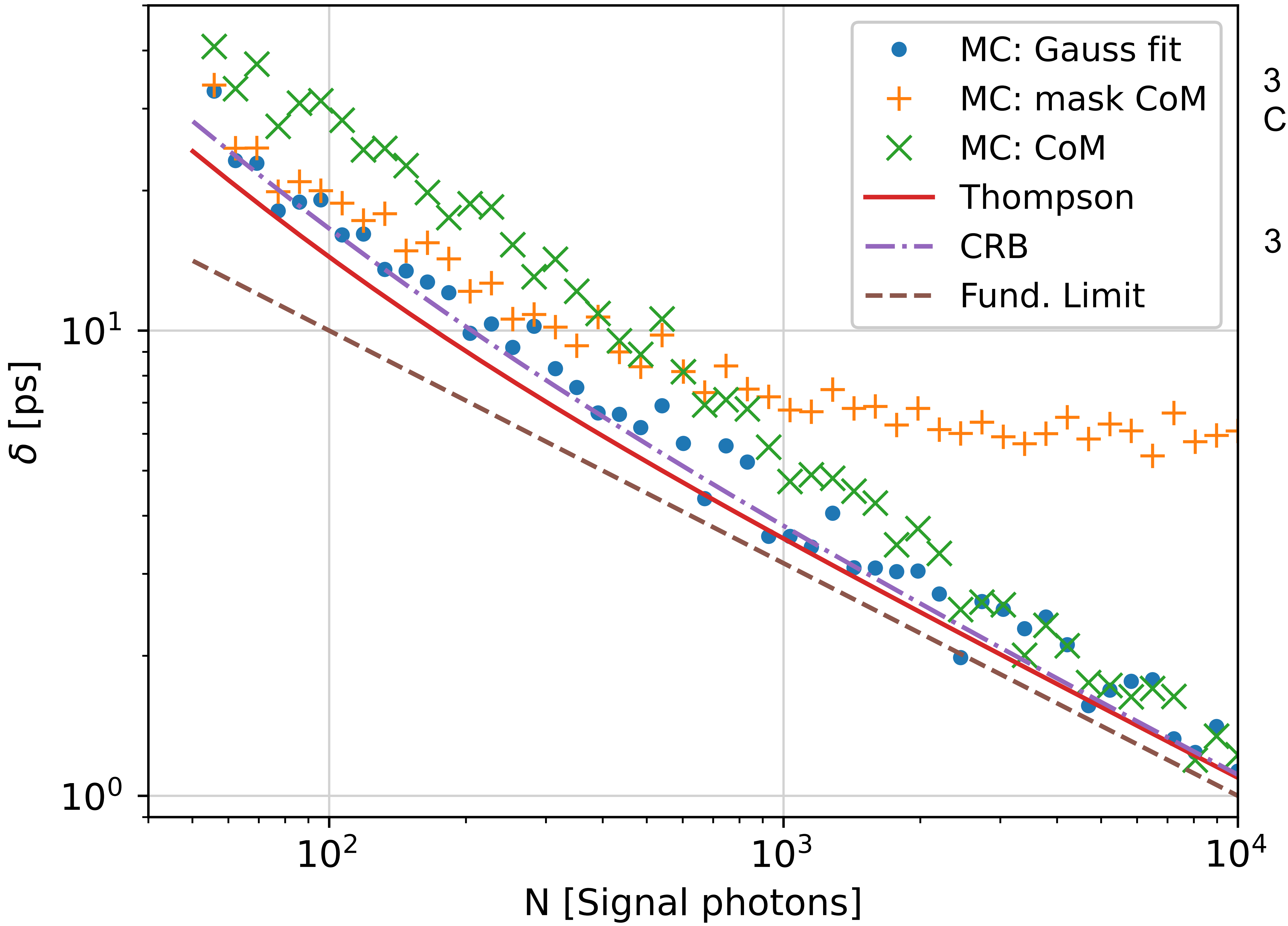
K. A. Winick, “Cramér–Rao lower bounds on the performance of charge coupled-device optical position estimators,” JOSA A, 1986. 10.1364/JOSAA.3.001809

L. J. Koerner, "Models of Direct Time-of-Flight Sensor Precision That Enable Optimal Design and Dynamic Configuration," 10.1109/TIM.2021.3073684

Rearrangement of 2 from: Gyongy, N. A. W. Dutton and R. K. Henderson, "Direct Time-of-Flight Single-Photon Imaging," IEEE Transactions on Electron Devices, 10.1109/TED.2021.3131430

Monte Carlo Model Verification



$$\sigma = 100\ ps,\ \ a = 1.5\sigma,\ \ b = 18.75$$

3 fitting methods
CoM = center-of-mass

3 models

L. J. Koerner, "Models of Direct Time-of-Flight Sensor Precision That Enable Optimal Design and Dynamic Configuration," 10.1109/TIM.2021.3073684

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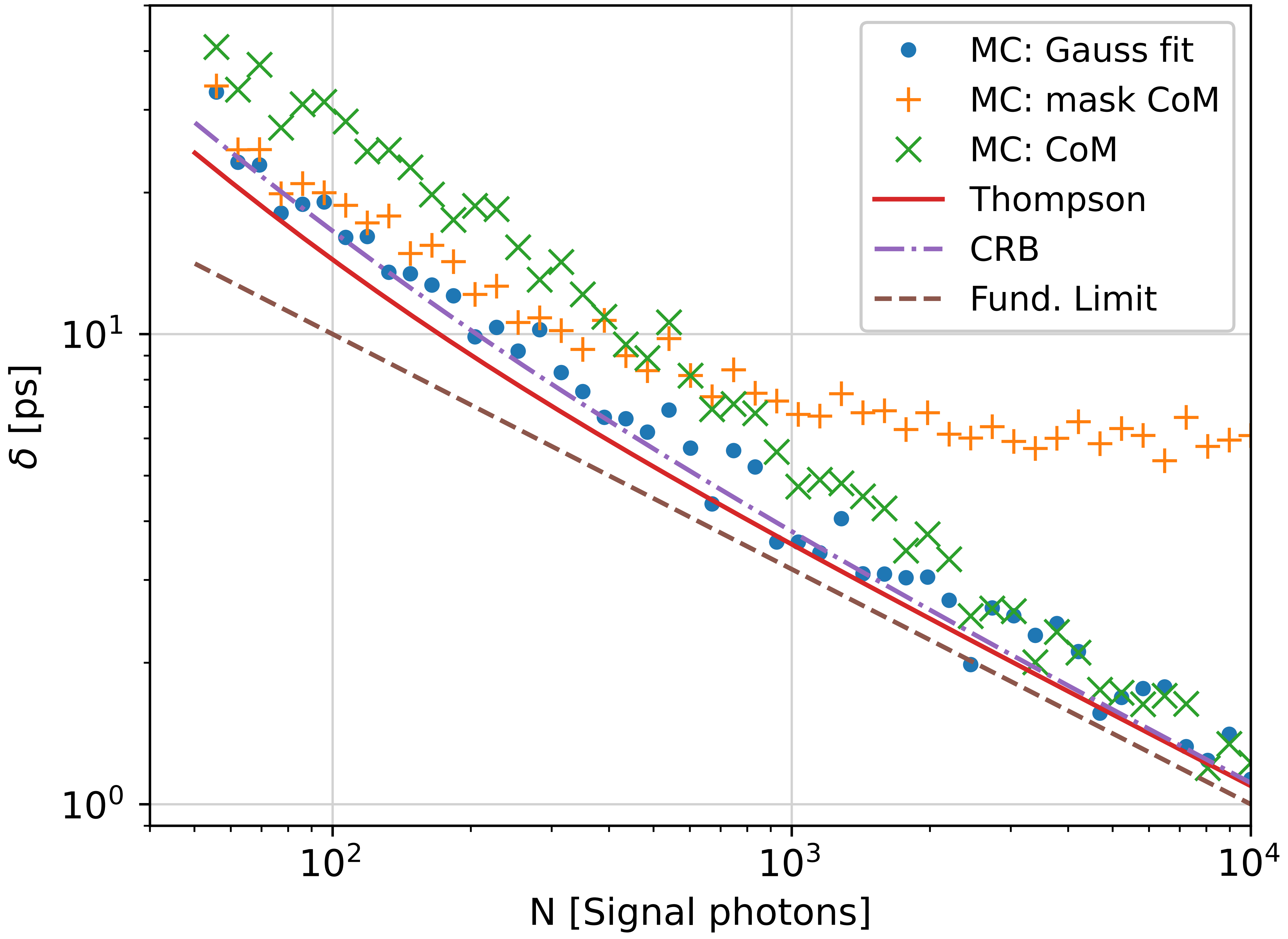

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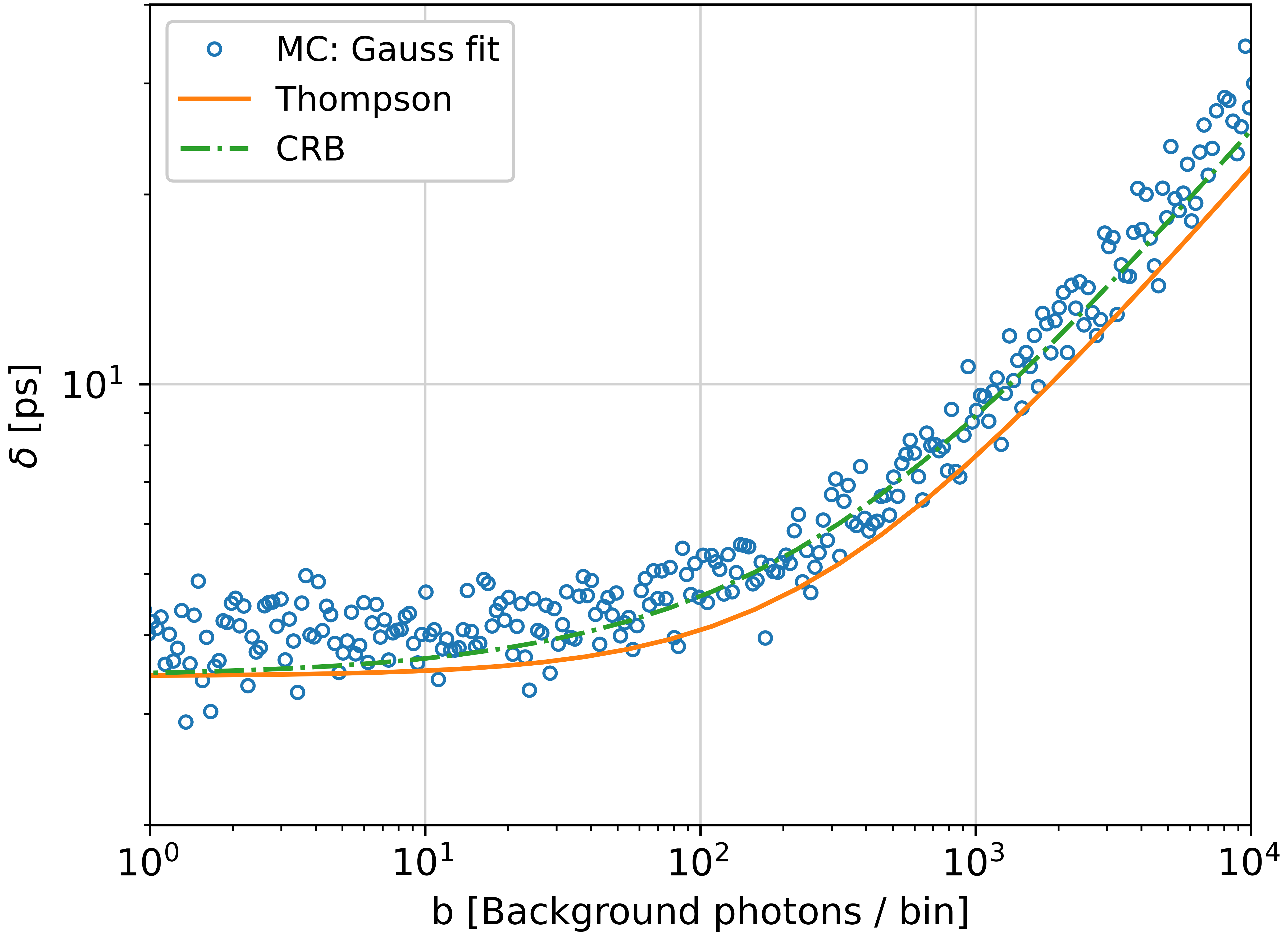

$\sigma = 100\ ps,\ \ a = 1.5\sigma,\ \ N = 1000$

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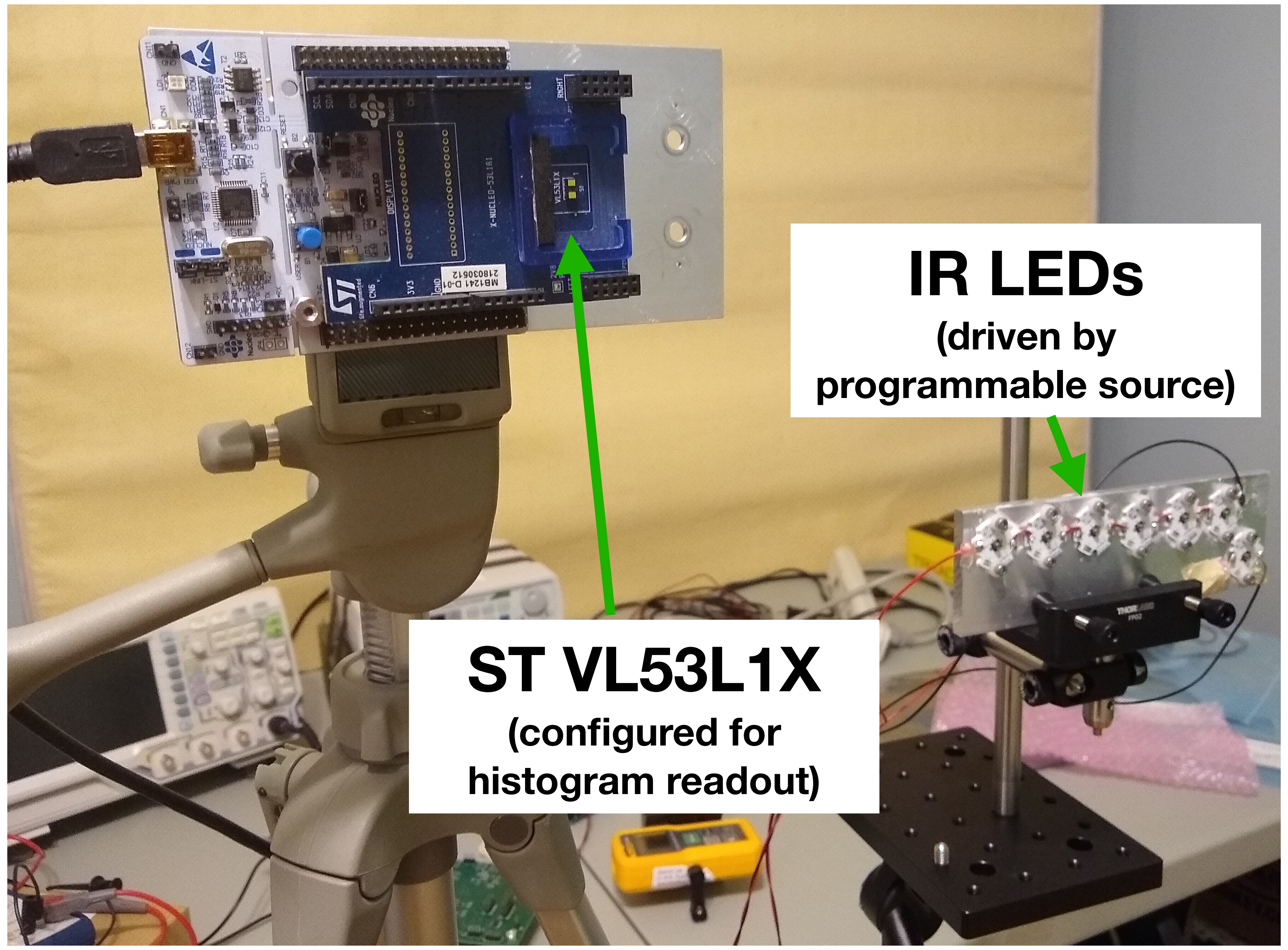
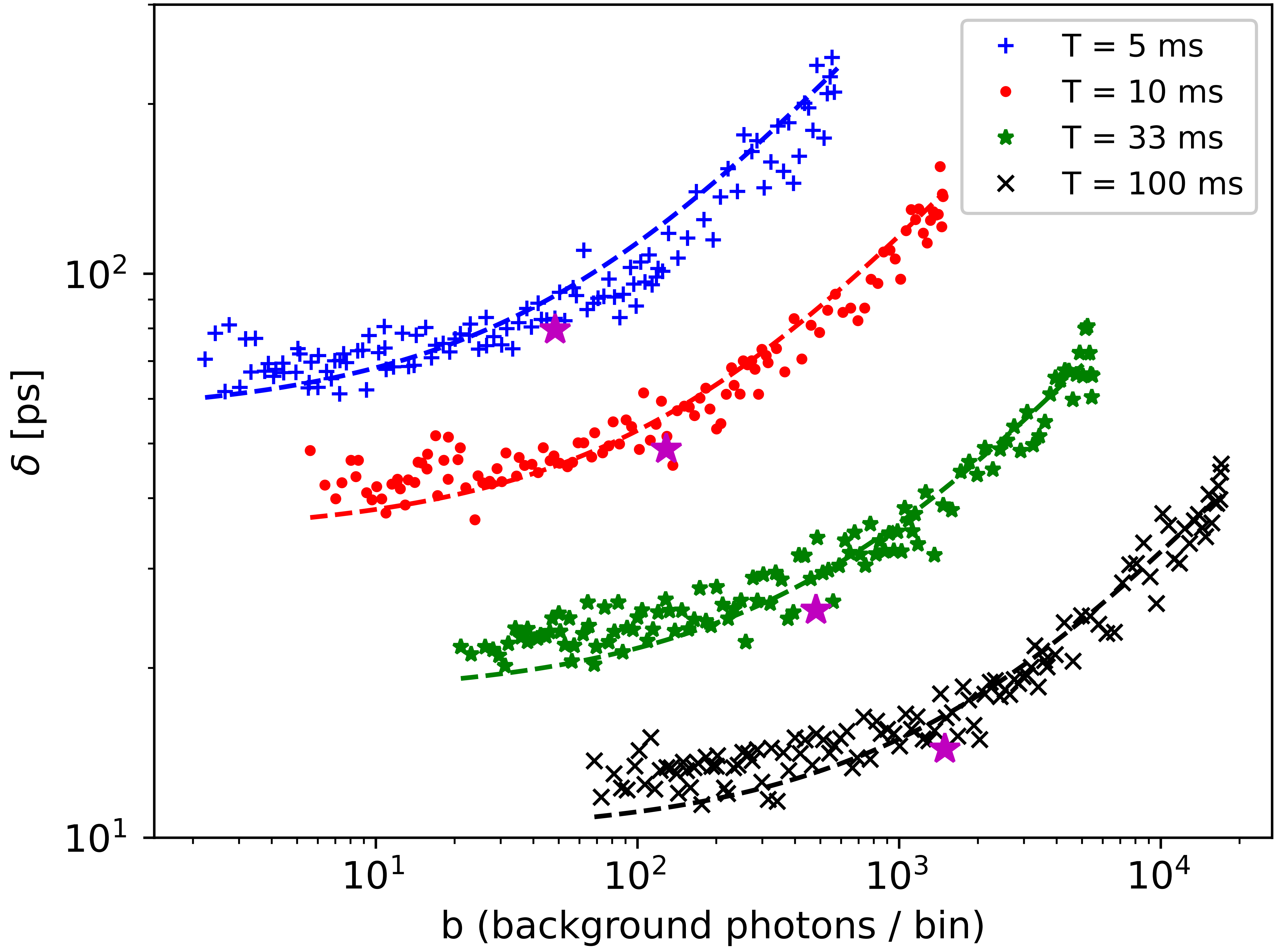
Experimental Validation



IR LEDs
(driven by programmable source)

ST VL53L1X
(configured for histogram readout)

- ST VL53L1X ToF sensor looking at gray(ish) wall. Configured for full histogram readout. Histograms processed (Gaussian fit) in host computer. 1.5 m object distance.
- IR LEDs used for adjustable background illumination.



★ Predicted point of $\sqrt{2}$ precision degradation as compared to b=0

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Analytical Model Takeaways

Symbol	Description
N	signal photons
b	background/bin
a	bin width
σ	timing jitter
δ	precision

1) Signal to background ratio

2) Coincidence detection

3) Regimes

4) TDC bin size

$$\delta = \frac{\sigma}{\sqrt{N}}\sqrt{1 + \frac{1}{12}\left(\frac{a}{\sigma}\right)^2 + 4\sqrt{\pi}\left(\frac{\sigma}{a}\right)\left(\frac{b}{N}\right)}$$

Analytical Model Takeaways

Symbol	Description
N	signal photons
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1) Let’s precisely define signal to background ratio.

$$\boxed{SBR = \frac{a}{\sigma}\frac{N}{b}}$$

Where b is the background counts / bin **within** the signal peak.

Gyongy, N. A. W. Dutton and R. K. Henderson, "Direct Time-of-Flight Single-Photon Imaging," 10.1109/TED.2021.3131430

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1) Let’s precisely define signal to background ratio.

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Where b is the background counts / bin **within** the signal peak.

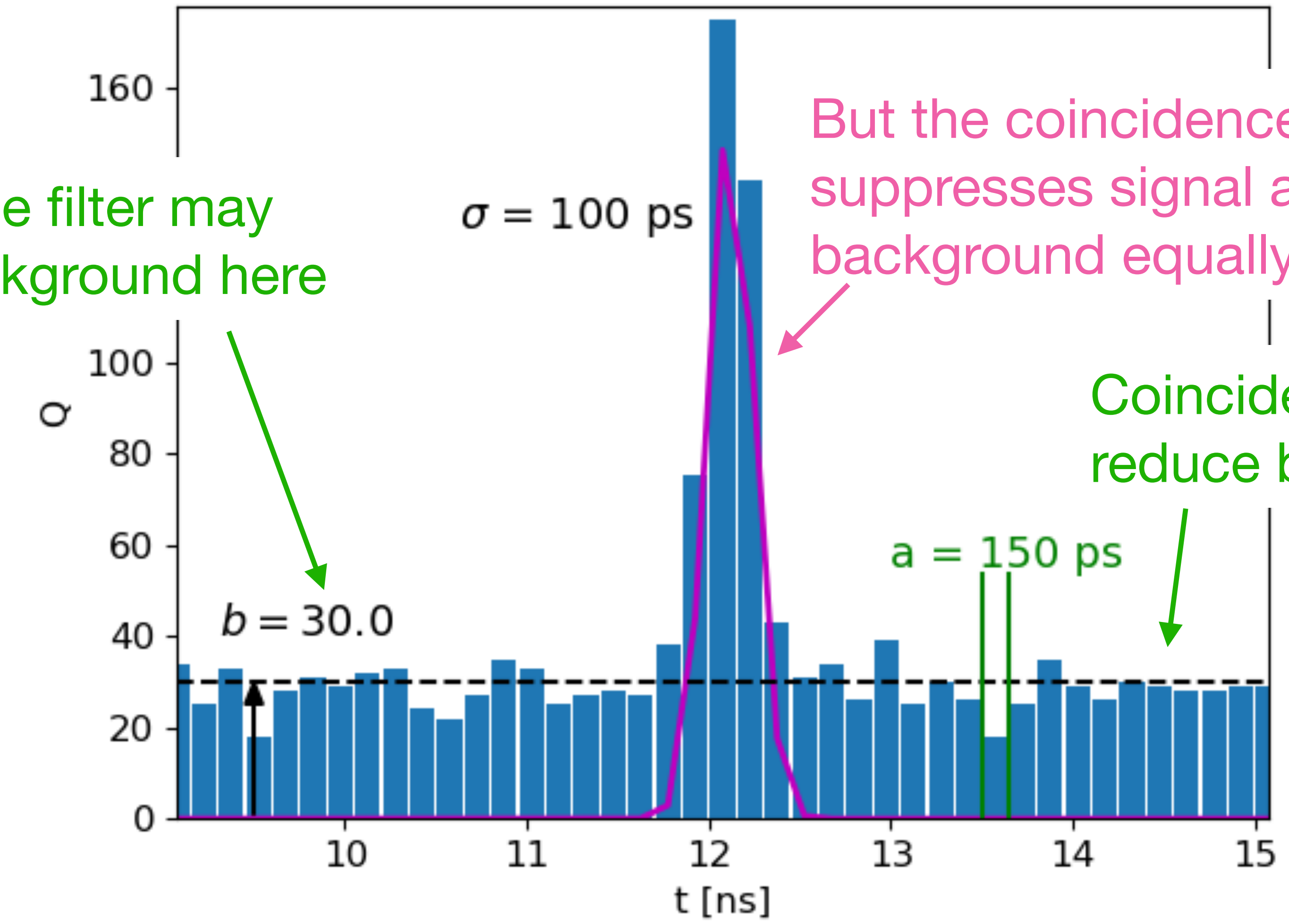
2) Coincidence detection may help prevent TDC pileup but otherwise degrades precision.

Collecting signal photons (N) is more “valuable” than rejecting background photons (b). As a non-linear rate filter, a coincidence circuit cannot distinguish between signal/background.

$$\boxed{\sqrt{4\sqrt{\pi}\left(\frac{\sigma^3}{a}\right)\left(\frac{b}{N^2}\right)}}$$

Background term

$$\delta = \frac{\sigma}{\sqrt{N}}\sqrt{1 + \frac{1}{12}\left(\frac{a}{\sigma}\right)^2 + 4\sqrt{\pi}\left(\frac{\sigma}{a}\right)\left(\frac{b}{N}\right)}$$



Coincidence filter may reduce background here

But the coincidence filter suppresses signal and background equally at the peak

Coincidence filter may reduce background here

Analytical Model Takeaways

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3) Regimes

Signal photon limited: $\delta \sim \frac{1}{\sqrt{N}}$

Background limited: $\delta \sim \frac{1}{N}$

$$\delta = \frac{\sigma}{\sqrt{N}}\sqrt{\underset{\text{signal limited}}{\underline{1 + \frac{1}{12}\left(\frac{a}{\sigma}\right)^2}} + \underset{\text{background}}{\underline{4\sqrt{\pi}\left(\frac{\sigma}{a}\right)\left(\frac{b}{N}\right)}}}$$

Equal contribution at $\frac{N}{b} = \frac{48\sqrt{\pi}\sigma^3}{12\sigma^2 a + a^3}$ **which for** $a = 2\sigma$ **is** $\frac{N}{b} = 2.66$

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signal limited

background

Signal photon limited: $\delta \sim \frac{1}{\sqrt{N}}$

Background limited: $\delta \sim \frac{1}{N}$

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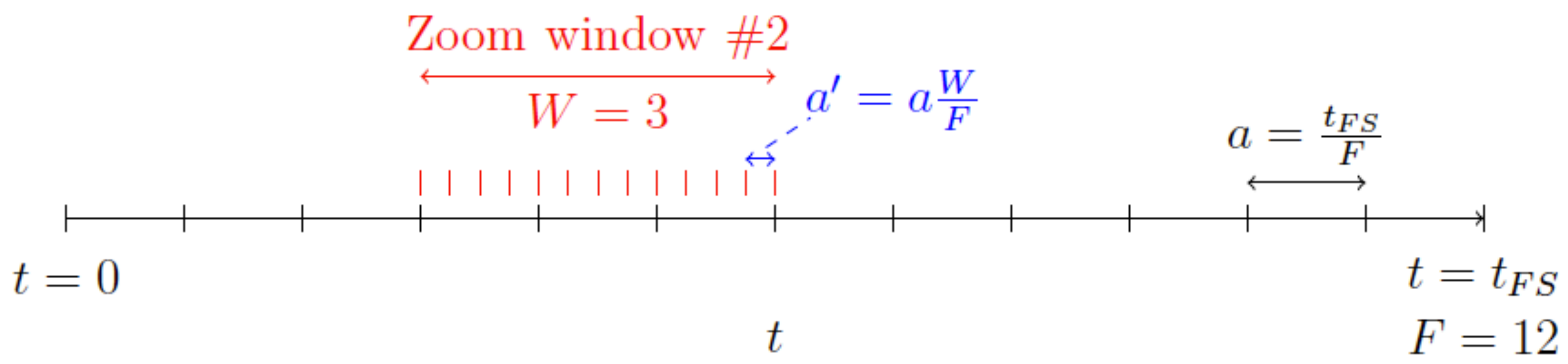
4) TDC bin size of $a \sim 1.5\sigma$ is good enough

$$\delta \sim \sqrt{1 + \frac{1}{12}\left(\frac{a}{\sigma}\right)^2 + \dots}$$

Optimal Zooming and Limiting Centroid Bias

Optimal Zooming for Precision

- Sensor model:
  - Fixed number of TDC bins (F) that zoom in temporally
  - Laser pulses distributed evenly among the zoom windows (# of windows = F/W)



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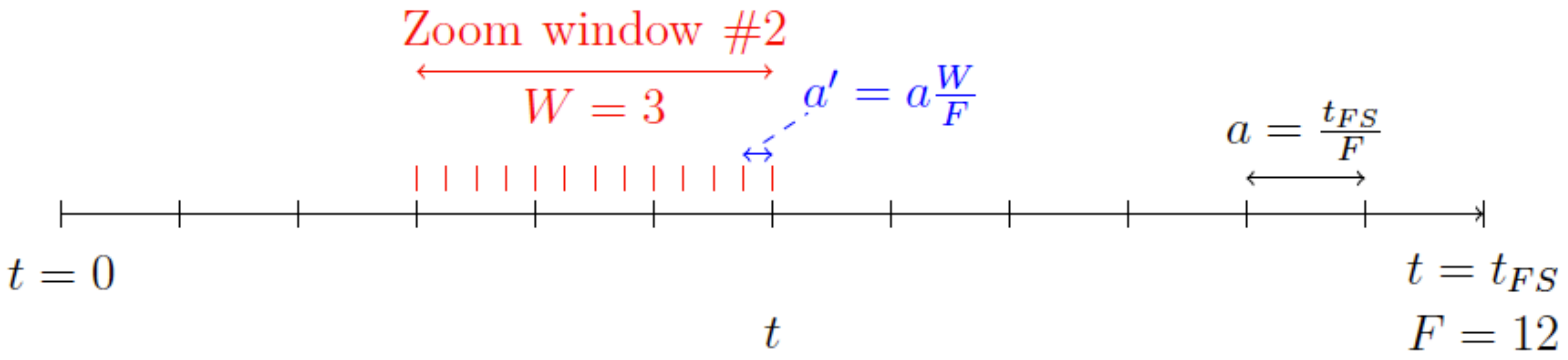


- Pile-up model:
    - SPADs disabled outside of the zoom window to reduce pile-up
    - Signal & background photons detected at the peak are reduced by e^{-bW}
    - Signal arrives at the end of the window (worst case for pile-up)

$$N' = N\left(\frac{W}{F}\right)e^{-bW}$$

$$b' = b\left(\frac{W}{F}\right)e^{-bW}$$

parameters are per pulse

Optimal Zooming for Precision

- Use N’ and b’ and differentiate precision with respect to W to find precision minimum

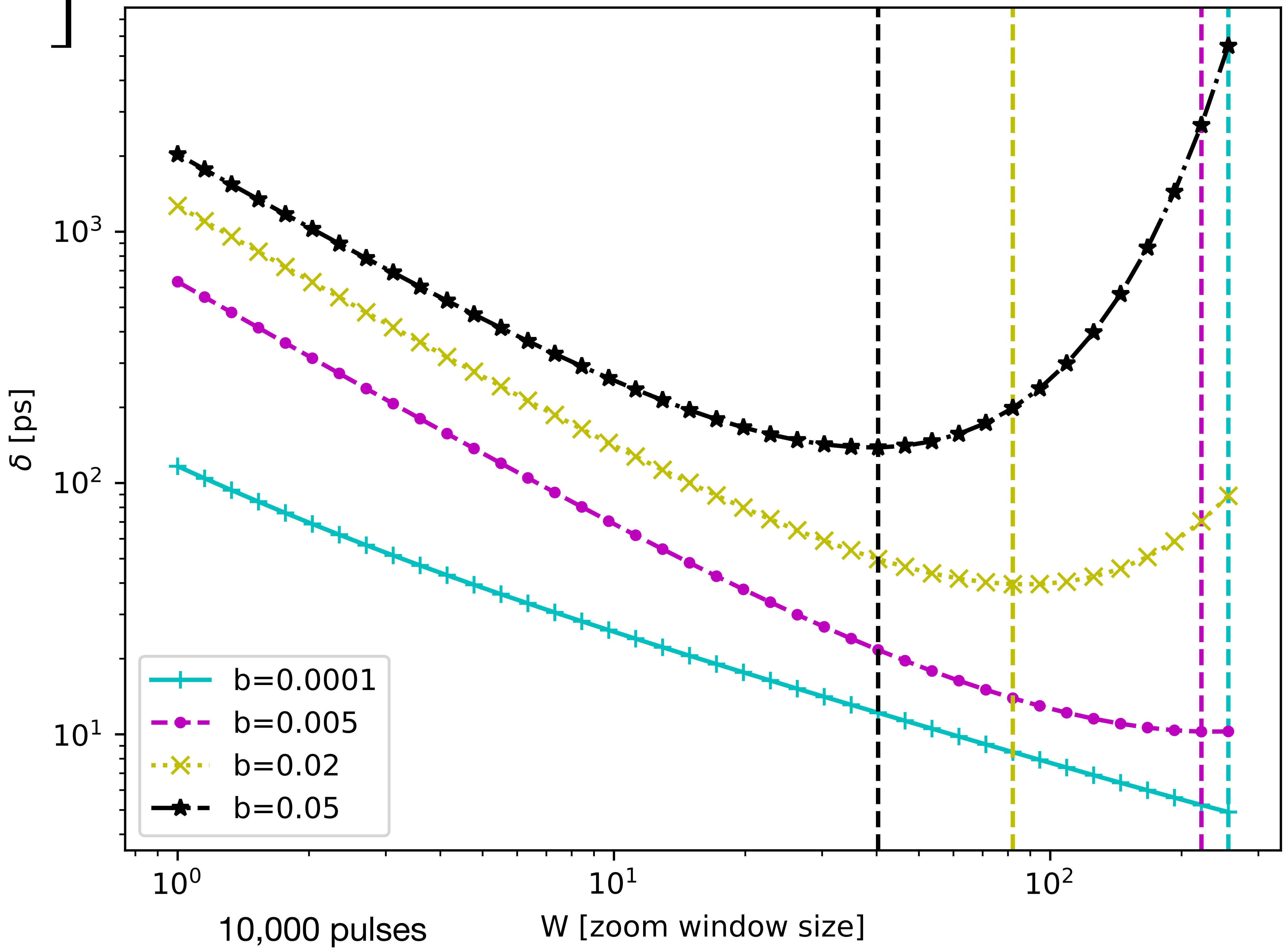
$$\frac{d\delta'^2}{dW} = \frac{F}{WN}e^{Wb}\left[\frac{\sigma^2\left(Wb-1\right)}{W} + \frac{Wa^2\left(Wb+1\right)}{12F^2} + \frac{4\sqrt{\pi}Fb\sigma^3\left(Wb-2\right)}{NW^2a}\right]$$

$$N' = N\left(\frac{W}{F}\right)e^{-bW}$$

$$b' = b\left(\frac{W}{F}\right)e^{-bW}$$

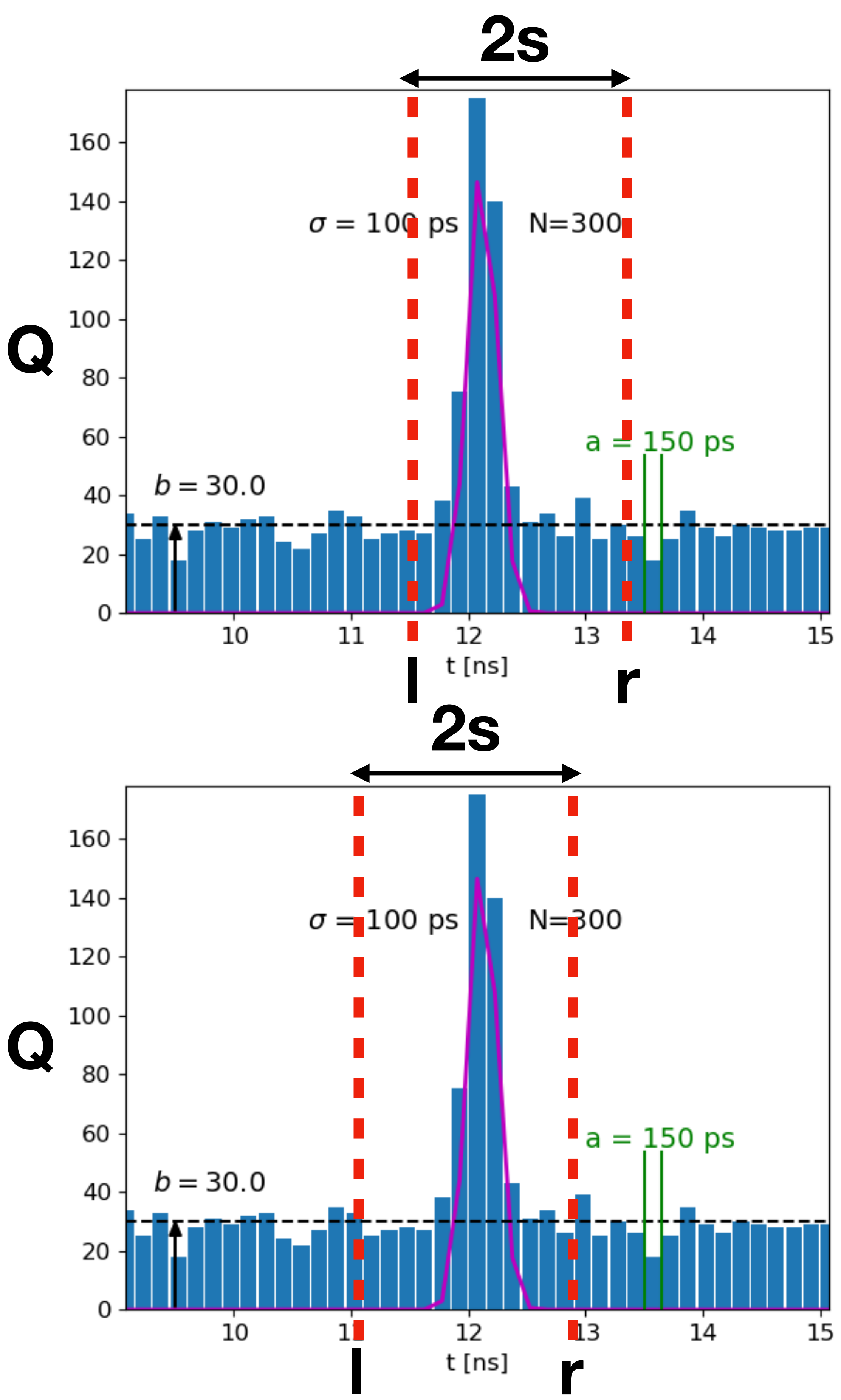
$$a' = a\frac{W}{F}$$

1. IRF and signal photon limited
   - Zoom to $W = 1/b$
2. TDC bin size limited
   - Zoom until the bin resolution is no longer limiting
3. Background limited
   - Zoom to $W = 2/b$



Centroid Bias

- Centroid is biased toward the window center in the presence of a uniform background.
- At modest background levels bias becomes more significant than precision.



$$\hat{t}_0{}' = \frac{\sum_{i=l}^{r} t_i(I_i + b)}{\sum_{i=l}^{r} (I_i + b)}$$

bias-free

$$\hat{t}_0 = \frac{\sum_{i=l}^{r} t_i I_i}{\sum_{i=l}^{r} I_i}$$

$Q_i = I_i + b$

I_i : signal photons in bin i

N	b	offset [bins]	Max bias [bins]	δ [bins]
1000	0	1	0.000	0.023
1000	10	1	0.037	0.023
1000	100	1	0.222	0.027
1000	1000	1	0.444	0.051

$a = 1.5\sigma, \ \ s = 3$

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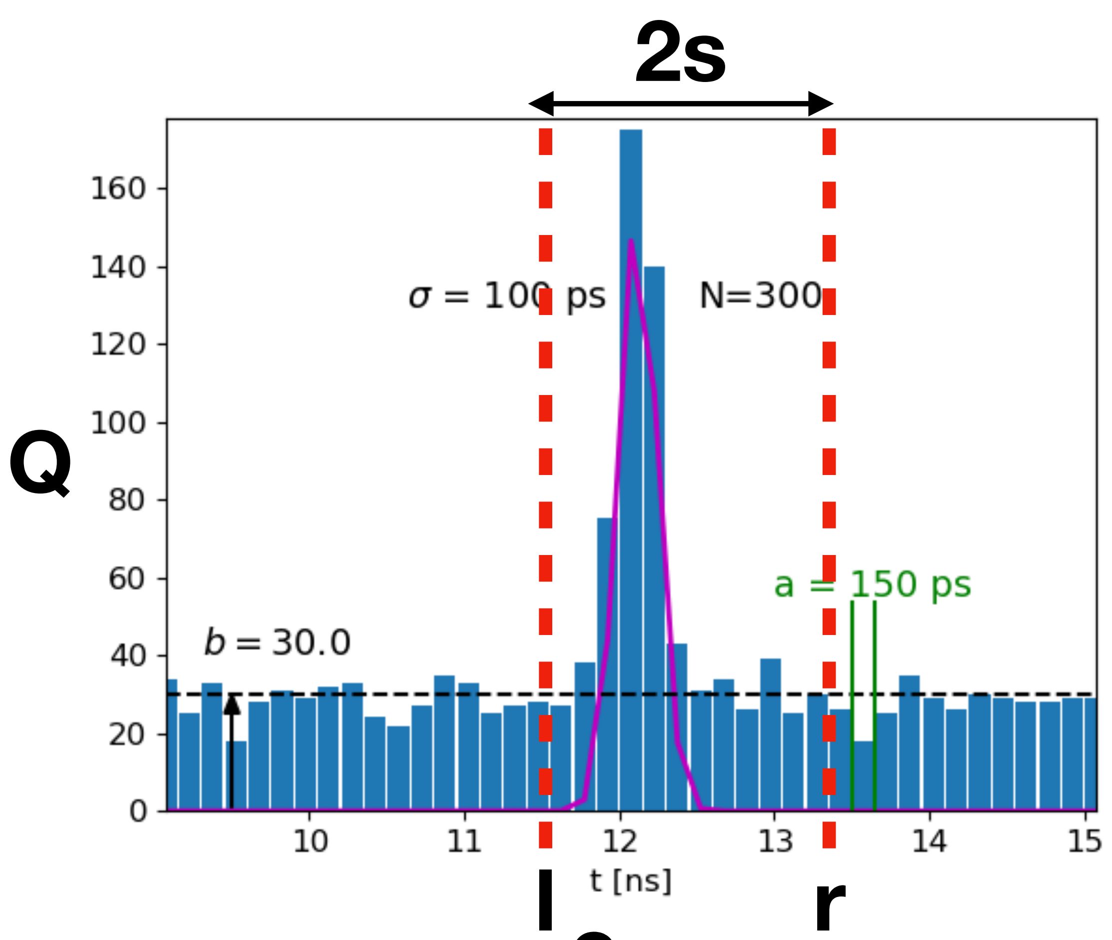

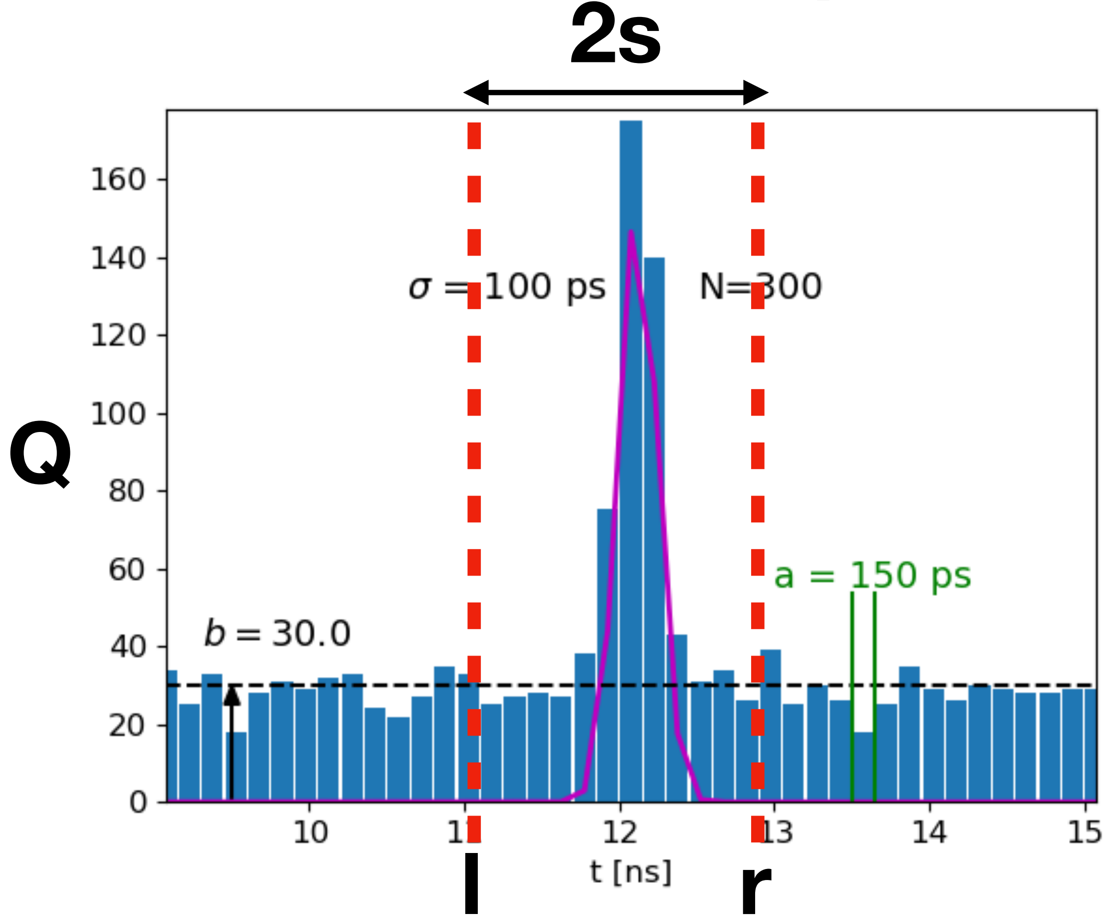

$$\hat{t}_0^{\,\prime} = \frac{\sum_{i=l}^{r} t_i(I_i + b)}{\sum_{i=l}^{r} (I_i + b)}$$

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$$\hat{t}_0^{\,\prime} = \frac{X}{N} + \frac{b(2s+1)(N(l+r) - 2X)}{2N(2bs + b + N)}$$

$$X = \sum_{i=l}^{r} t_i I_i$$

$$N = \sum_{i=l}^{r} I_i$$

bias-free

bias

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Centroiding to Minimize Bias

- To minimize centroid bias:
  1. Limit window span
  2. Remove background by subtracting or thresholding
- But (local) background may not be known or may change with time
- Alternative approach:
  3. Iterative centroid using a shifting (sub-bin) window [Berglund 2008].

$$\hat{t}_0{}' = \underbrace{\frac{X}{N}}_{\text{bias-free}} + \underbrace{\frac{b(2s+1)(N(l+r)-2X)}{2N(2bs+b+N)}}_{\text{bias}}$$

$$X = \sum_{i=l}^{r} t_i I_i$$

$$N = \sum_{i=l}^{r} I_i$$

Berglund, Andrew J., et al. "Fast, bias-free algorithm for tracking single particles with variable size and shape." Optics express 10.1364/OE.16.014064

Iterative Centroid (CoM) to Minimize Bias

- Alternative approach: iterative centroid using a shifting (sub-bin) window [Berglund 2008].

$$\hat{t}_{0_{n+1}} = \frac{\sum_{i=l_n+1}^{r_n-1} t_i Q_i + (0.5 - \Delta_n) t_{c-s} Q_{c-s} + (0.5 + \Delta_n) t_{c+s} Q_{c+s}}{\sum_{i=l_n+1}^{r_n-1} Q_i + (0.5 - \Delta_n) Q_{c-s} + (0.5 + \Delta_n) Q_{c+s}}$$

A fraction of the leftmost bin

A fraction of the rightmost bin

$$\begin{aligned} 2s &= r - l \\ c &= \text{argmin}(\hat{t}_{0_n} - t_i) \\ l_{n+1} &= c - s \\ r_{n+1} &= c + s \\ \Delta_{n+1} &= (\hat{t}_{0_n} - t_c)/\Delta t \end{aligned}$$

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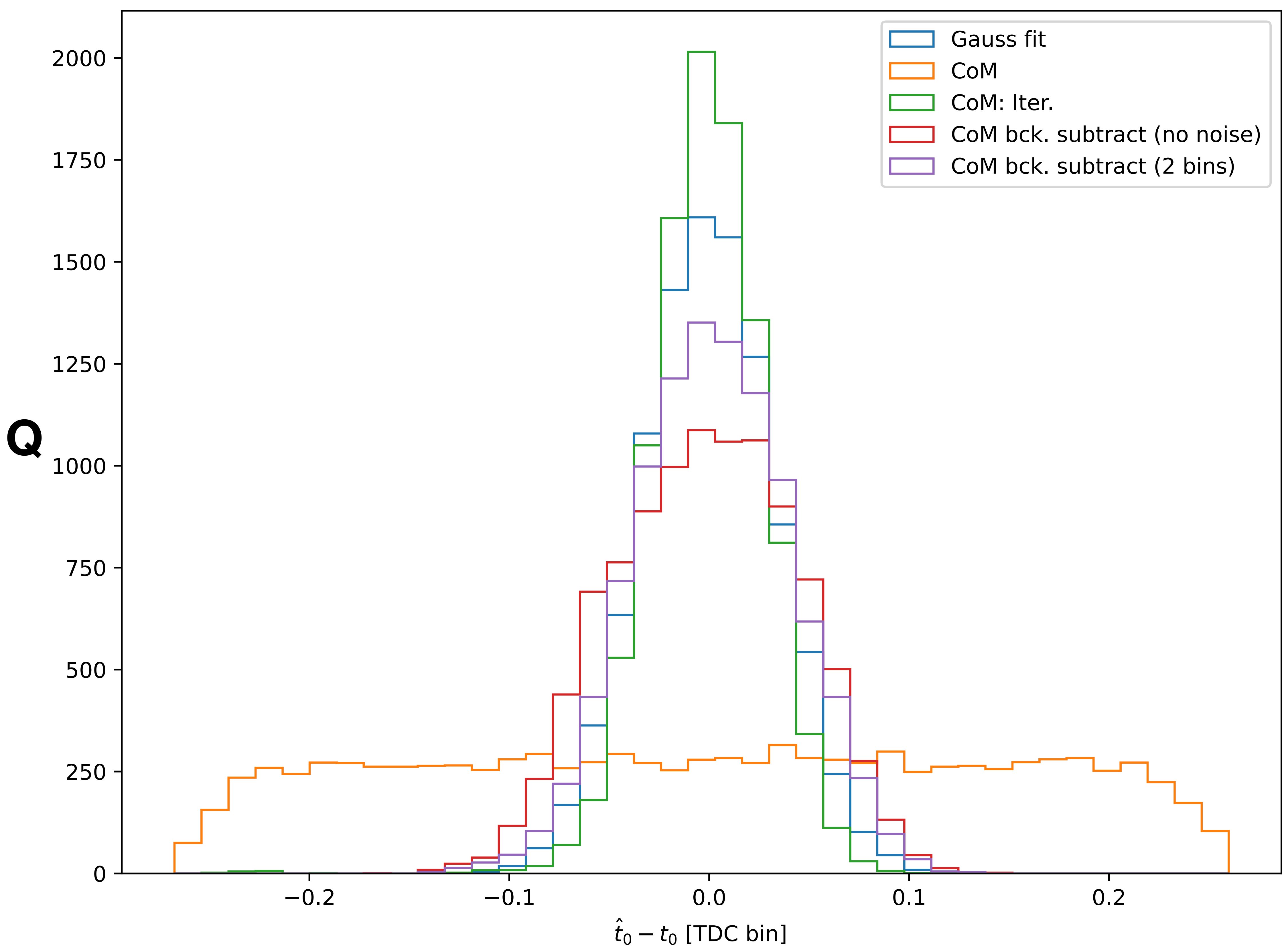
Iterative Centroid (CoM) to Minimize Bias

- Alternative approach: iterative centroid using a shifting (sub-bin) window [Berglund 2008].
- Monte Carlo simulations: to investigate bias with different localization methods



$a = 1.5\sigma, \ b = 1000, N = 10000, s = 3$

Error in estimate

Acknowledgements, Future Work, and Collaborations

- Thanks to the UST School of Engineering and undergraduate student Ryan Jans
- Collaborations are important to our work. Please reach out!
- Future investigations to co-design/optimize ToF sensors for science measurements (non-depth).

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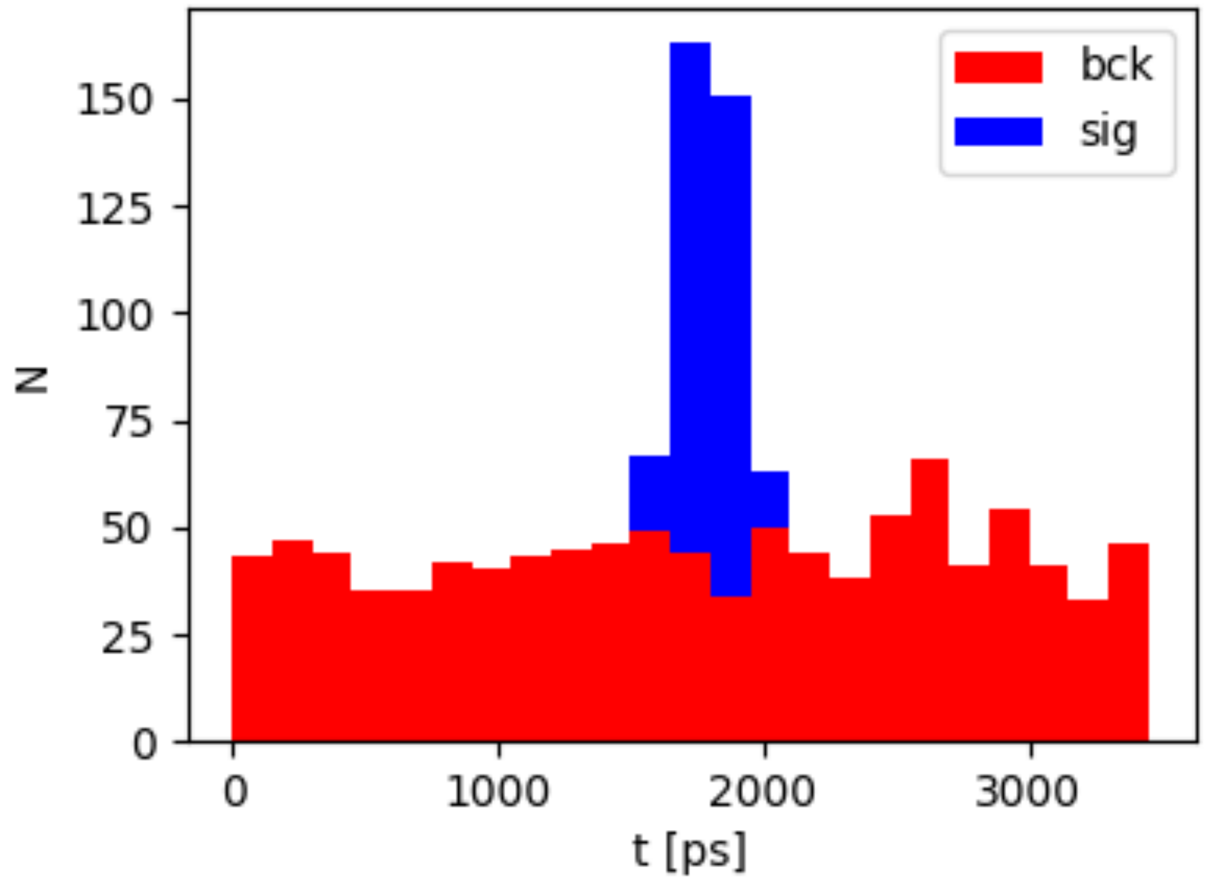
Backup Slides

Coincidence Detection Simulations

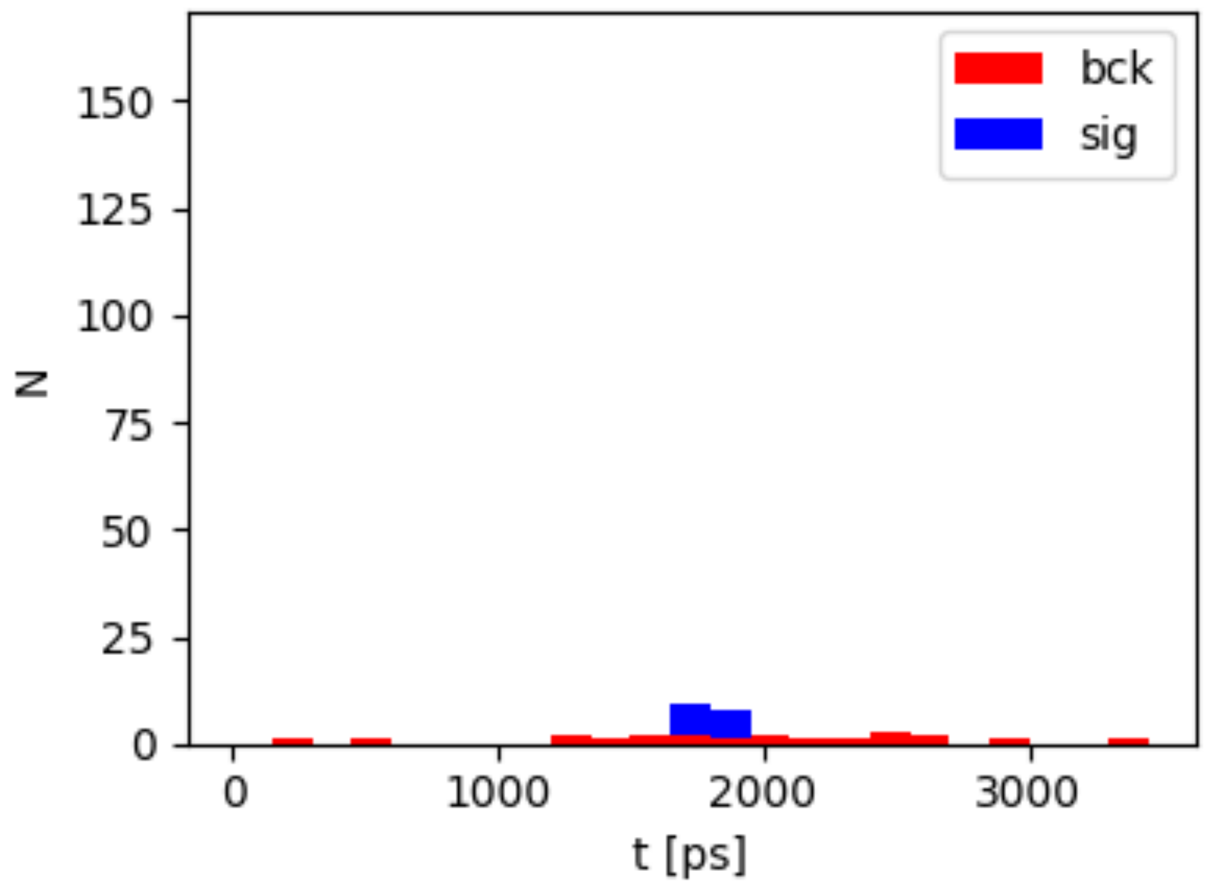
Monte Carlo simulations. 10,000 laser pulses, IRF 1σ = 100 ps, TDC bin = 150 ps, 24 TDC bins; 16 pixel macro-pixel (ignore dead-time effects).
Uniform background, Poisson distributed arrival times. 400 ps coincidence window; coincidence depth of 2.
Signal centered at 1800 ps, Poisson distributed photon number, Gaussian spread of arrival times.

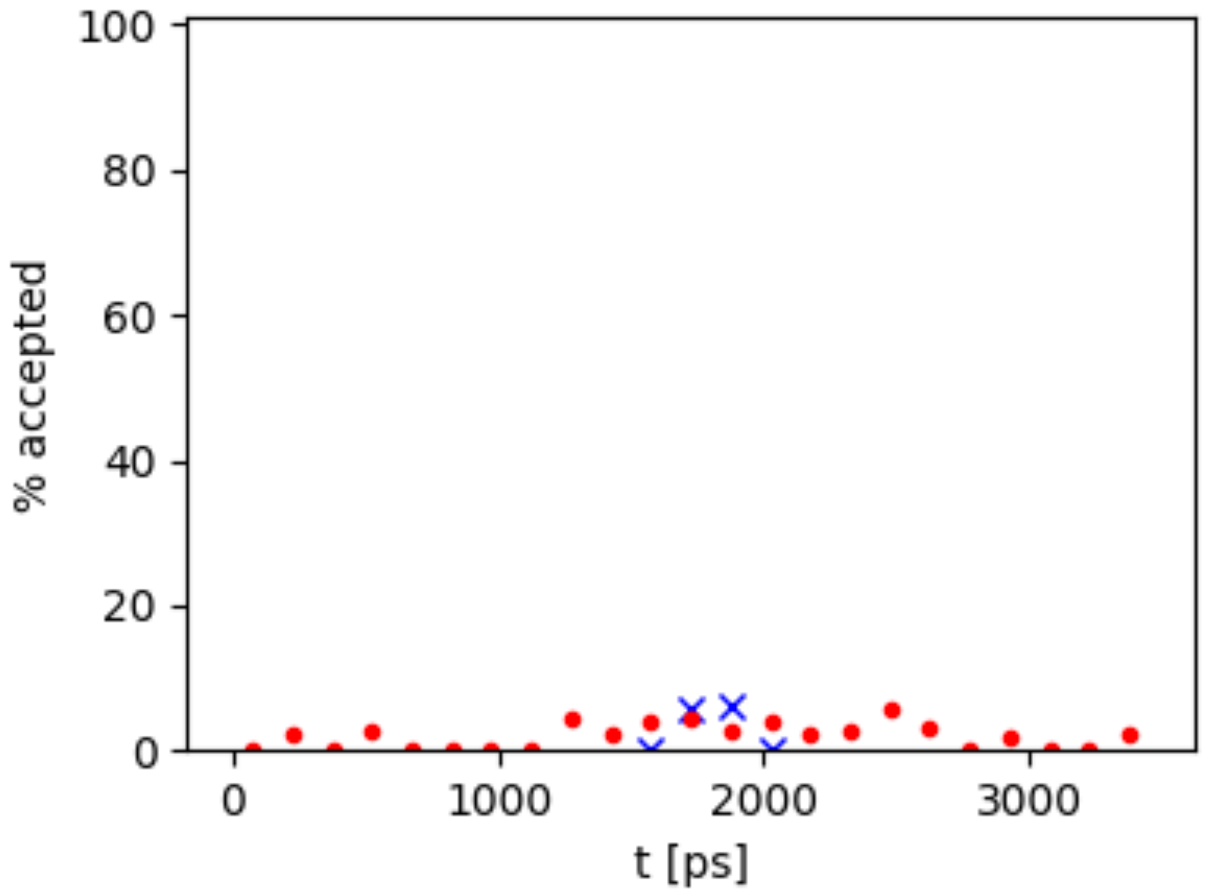
$$\lambda_c = \lambda e^{-\lambda t_{win}}$$

No coincidence



Coincidence





Photons accepted

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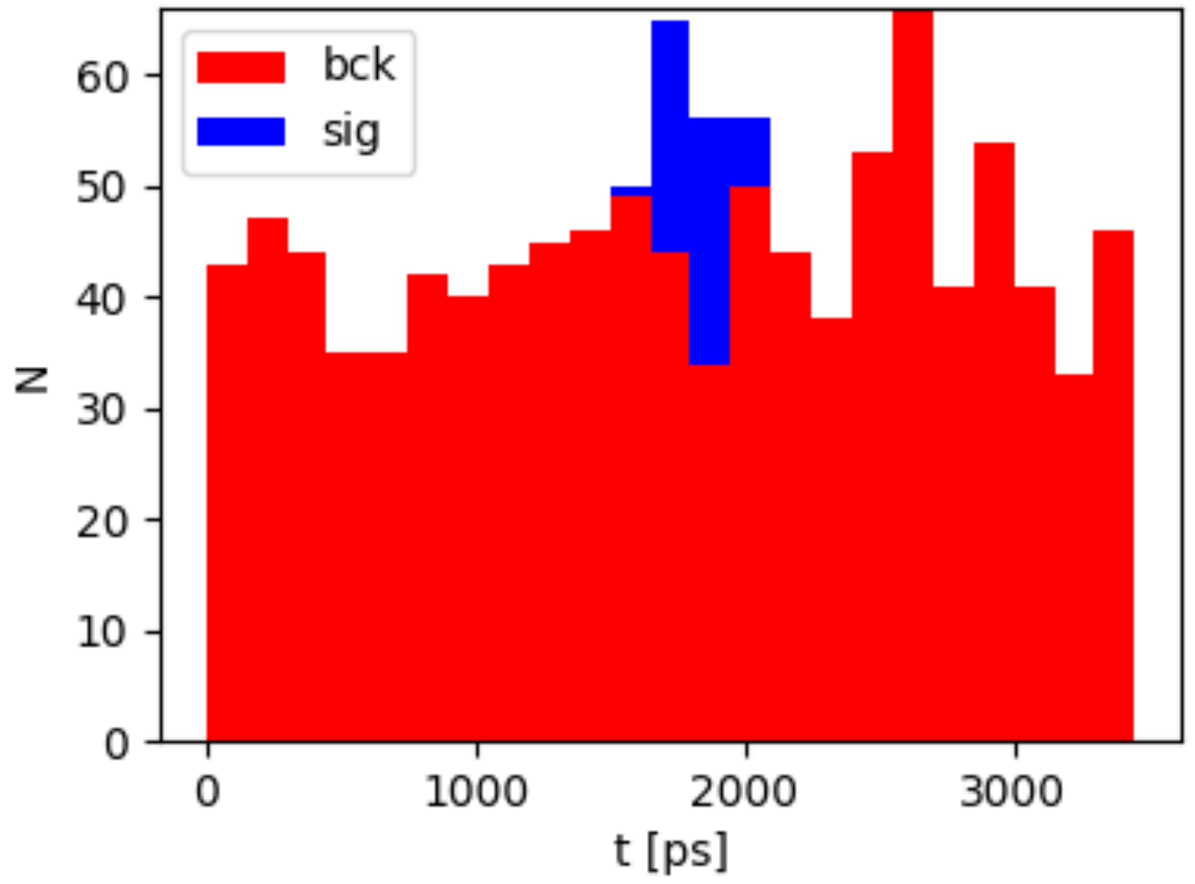

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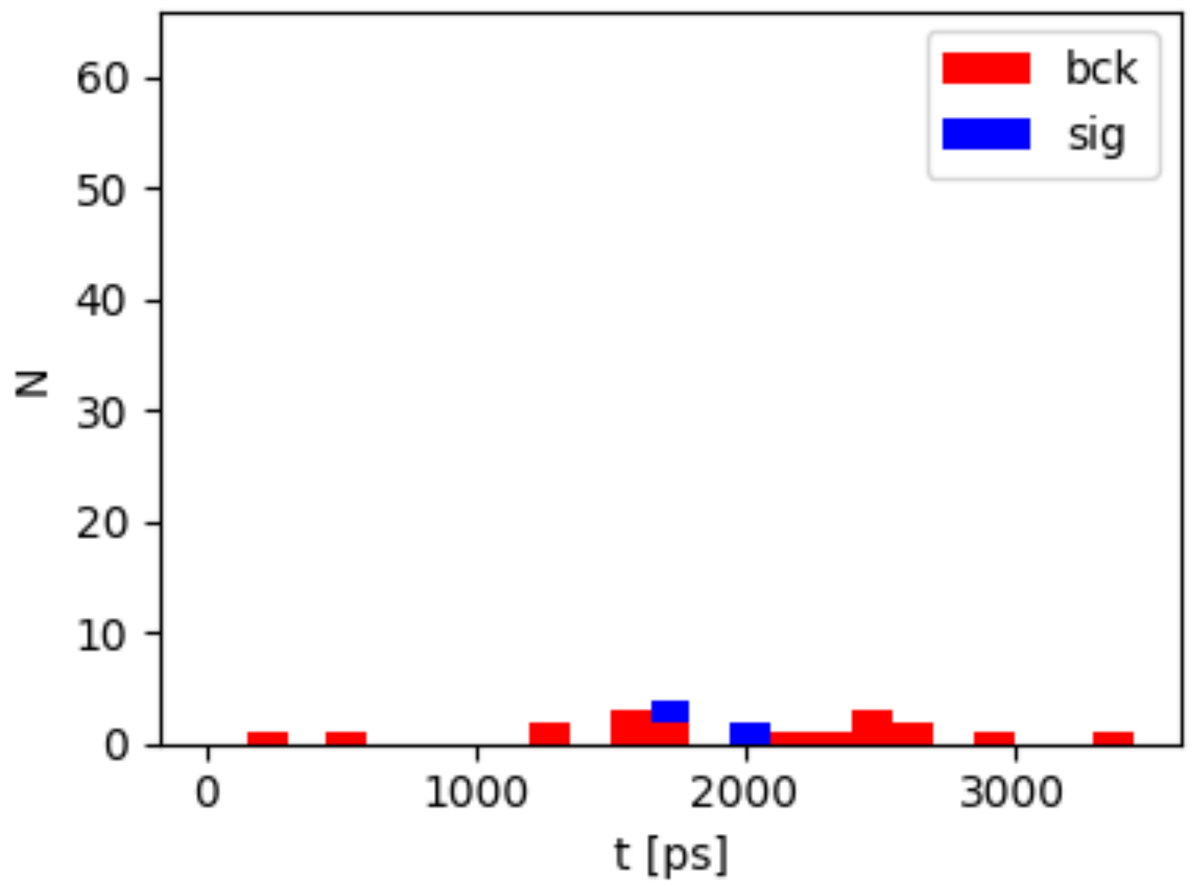
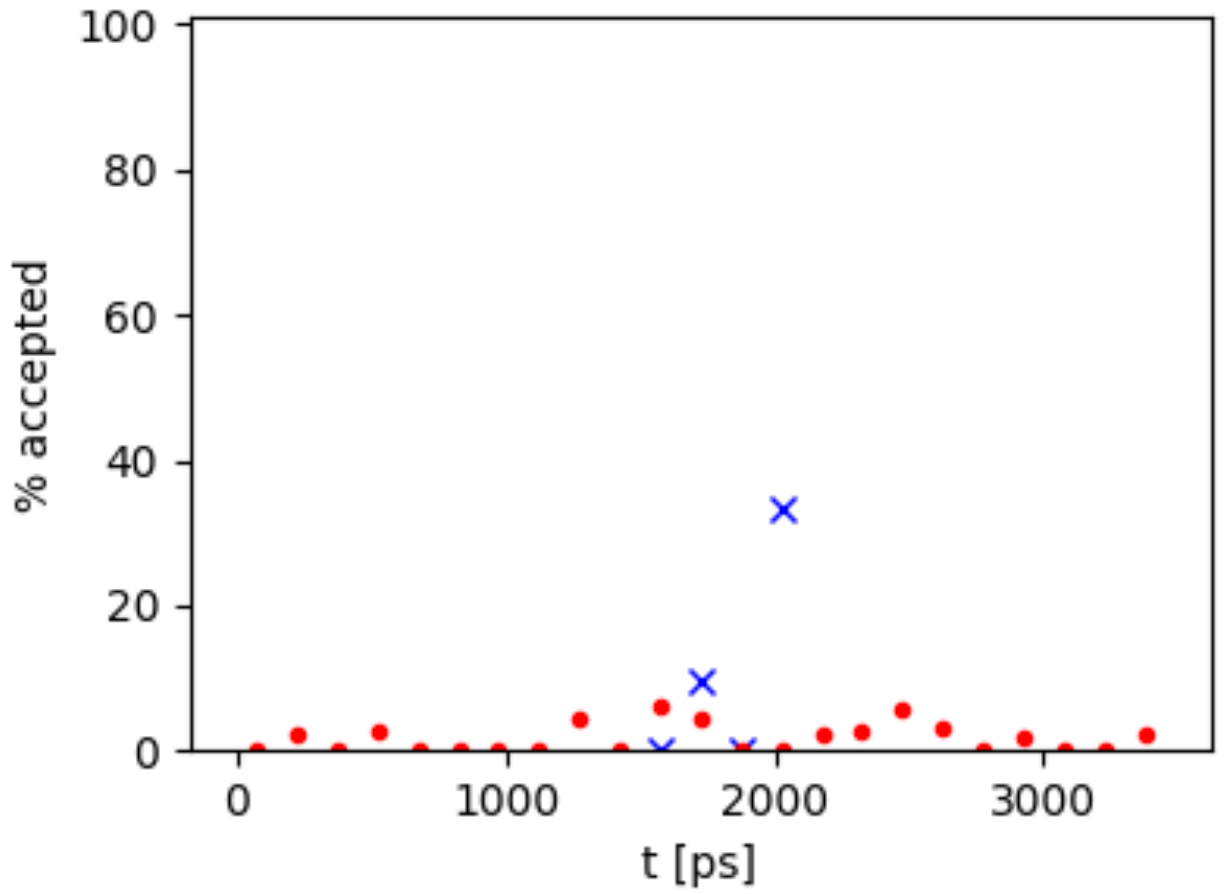




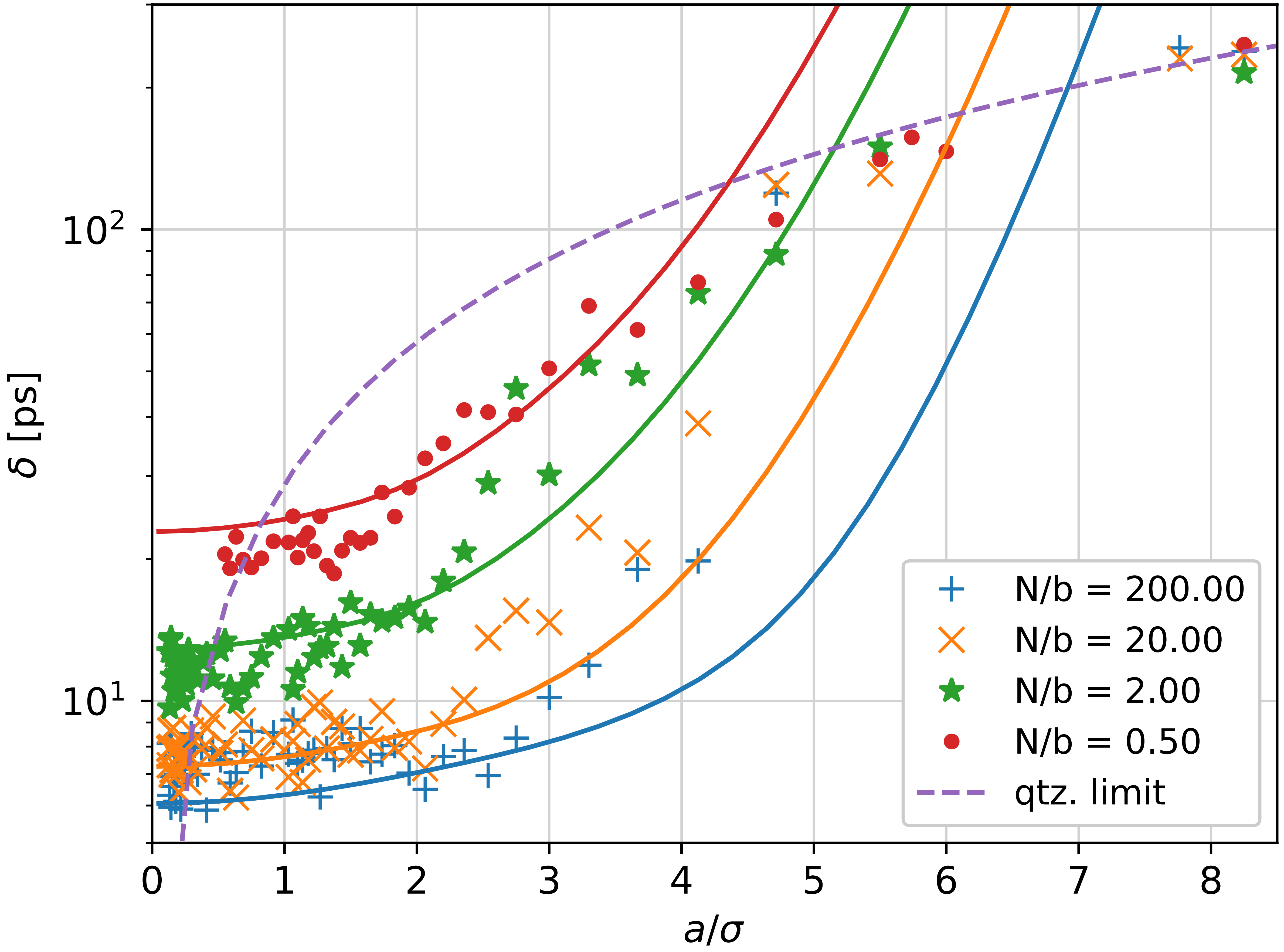
Photons accepted

Optimal TDC Bin Size

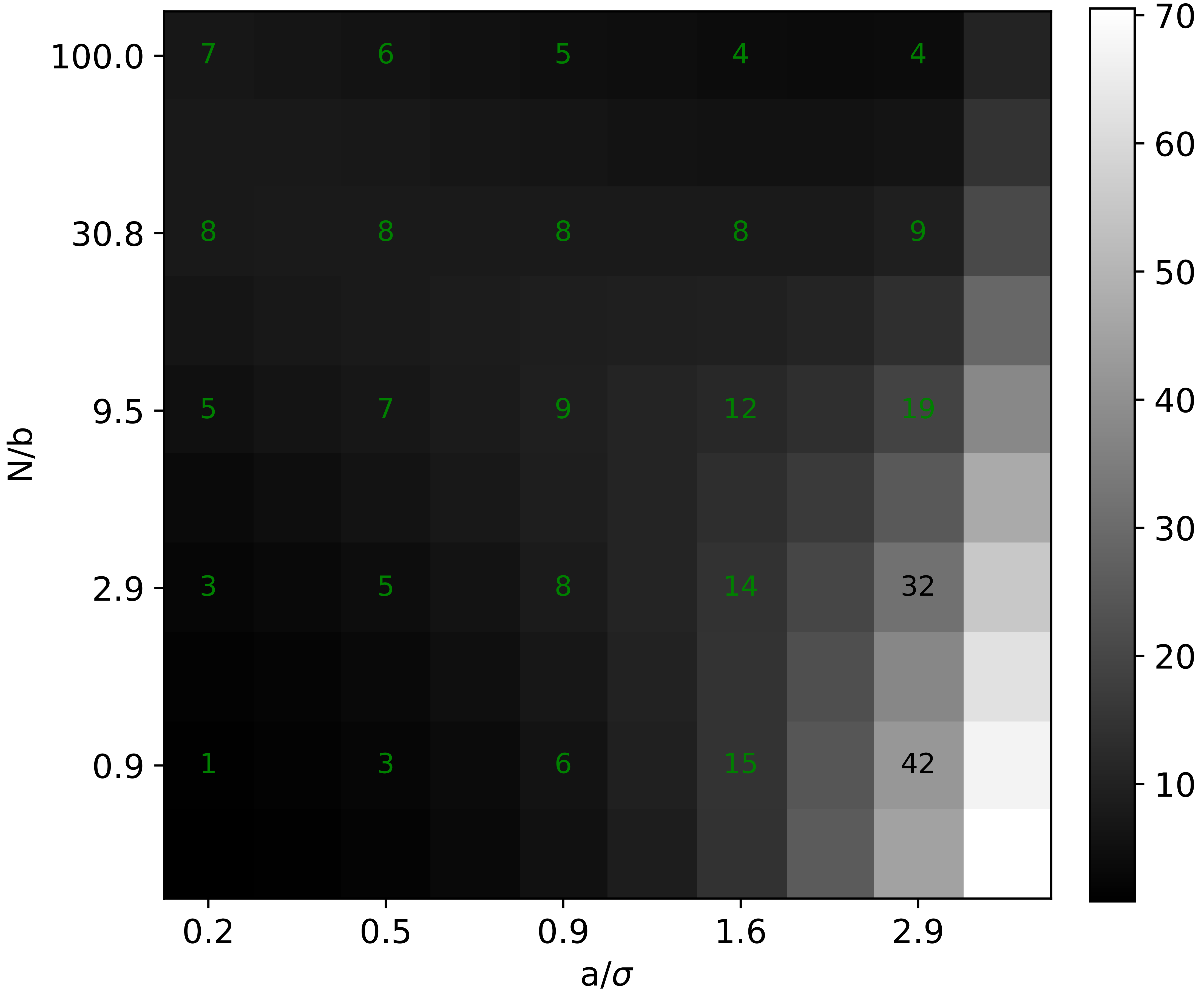
TABLE I

TDC BIN RESOLUTION (a) IN TERMS OF THE TEMPORAL SPREAD (σ) THAT PRODUCES A GIVEN PRECISION DEGRADATION FOR VARIOUS LEVELS OF SNR (N/b). THE PRECISION AT $a = 0$ IS REPRESENTED AS δ_0 (NO DEGRADATION). THE SECOND COLUMN SHOWS THE a/σ VALUES FOR A 10% DEGRADATION OF PRECISION FROM THE ZERO TDC BIN WIDTH BASELINE, WHEREAS THE THIRD COLUMN SHOWS THE SAME FOR A 41% DEGRADATION

N/b ($a/\sigma = 1$)	a/σ at $\delta = 1.1\delta_0$	a/σ at $\delta = 1.41\delta_0$
200	1.55	3.22
20	1.47	2.78
2	1.35	2.38
0.5	1.31	2.26



Thompson vs. CRB



Percent difference between the CRB timing precision and the Thompson model as the SNR (N/b) and a/σ is varied. In all cases, the CRB precision estimate is greater than the Thompson model.